DERIVACE

Zderivujte funkce:

(a) $f(x) = (1 + 3x - 5x^2)^{30}$;

(b) $f(x) = \sqrt{1 - x^2}$;

(c) $f(x) = \dfrac{1 - \sqrt[3]{2x}}{1 + \sqrt[3]{2x}}$;

(d) $f(x) = \dfrac{\sqrt{2x^2 - 2x + 1}}{x}$;

(e) $f(x) = \dfrac{x^2}{\sqrt{x^2 + a^2}},\ a \neq 0$;

(f) $f(x) = \dfrac{x^3}{3\sqrt{(1 + x^2)^3}}$;

(g) $f(x) = (a^{\frac{2}{3}} - x^{\frac{2}{3}})^{\frac{3}{2}},\ \ a > 0$;

(h) $f(x) = \cos^2 x$;

(i) $f(x) = 3\sin^2 x - \sin^3 x$;

(j) $f(x) = \operatorname{tg} x - \frac{1}{3}\operatorname{tg}^3 x + \frac{1}{5}\operatorname{tg}^5 x$;

(k) $f(x) = \sin^2(\cos 3x)$;

(l) $f(x) = \sin\sqrt{1 + x^2}$;

(m) $f(x) = \arcsin\dfrac{1}{x^2}$;

(o) $f(x) = \arccos\dfrac{2x - 1}{\sqrt{3}}$;

(p) $f(x) = \frac{1}{2}\operatorname{arctg}\frac{x}{2} - \frac{1}{3}\operatorname{arctg}\frac{x}{3}$;

(q) $f(x) = \ln^2 x - \ln(\ln x)$;

(r) $f(x) = e^{\sin^2 x}$;

(s) $f(x) = 3^{\operatorname{cotg}\frac{1}{x}}$;

Výsledek:

(a) $30(3 - 10x)(1 + 3x - 5x^2)^{29}$;

(b) $\dfrac{-x}{\sqrt{1 - x^2}},\ x \in (-1, 1)$;

(c) $\dfrac{-4}{3\sqrt[3]{4x^2}(1 + \sqrt[3]{2x})^2}, x \neq -\frac{1}{2},\ 0$;

(d) $\dfrac{x - 1}{x^2\sqrt{2x^2 - 2x + 1}},\ x \neq 0$;

(e) $\dfrac{x(x^2 + 2a^2)}{\sqrt{(x^2 + a^2)^3}}$;

(f) $\dfrac{x^2}{\sqrt{(1 + x^2)^5}}$;

(g) $-\dfrac{\sqrt{a^{\frac{2}{3}} - x^{\frac{2}{3}}}}{\sqrt[3]{x}},\ x \in (-a, a)$;

(h) $-\sin 2x$;

(i) $\frac{3}{2}\sin 2x(2 - \sin x)$;

(j) $1 + \operatorname{tg}^6 x$, vyjádřete $\cos^2 x$ pomocí $\operatorname{tg} x, x \neq (2k + 1)\frac{\pi}{2}$;

(k) $-3\sin 3x \sin(2\cos 3x)$;

(l) $\dfrac{x\cos\sqrt{1 + x^2}}{\sqrt{1 + x^2}}$;

(m) $\dfrac{-2}{x\sqrt{x^4 - 1}},\ |x| > 1$;

(o) $-\dfrac{\sqrt{2}}{\sqrt{1 + 2x - 2x^2}},\ x \in \left(\dfrac{1 - \sqrt{3}}{2}, \dfrac{1 + \sqrt{3}}{2}\right)$;

(p) $\dfrac{5}{x^4 + 13x^2 + 36}$;

(q) $\dfrac{2\ln x}{x} - \dfrac{1}{x\ln x},\ x > 1$;

(r) $e^{\sin^2 x}\sin 2x$;

(s) $\dfrac{3^{\operatorname{cotg}\frac{1}{x}}\ln 3}{x^2\sin^2\dfrac{1}{x}},\ x \neq 0, \frac{1}{k\pi}, k \in \mathbf{Z} - \{0\}$;

Derivujte dané funkce:

$$f(x) = \frac{x}{\ln x} - \frac{1}{\sqrt[7]{x^2}}$$

$$g(x) = e^{x^2} \cdot \ln x$$

$$h(x) = \left(\sqrt{x}\right)^{\cos x}$$

$$\begin{bmatrix} f'(x) = \dfrac{\ln x - 1}{\ln^2 x} + \dfrac{2}{7\sqrt[7]{x^9}} \\ g'(x) = e^{x^2}\left(2x\ln x + \dfrac{1}{x}\right) \\ h'(x) = \left(\sqrt{x}\right)^{\cos x} \cdot \left(-\sin x \ln\sqrt{x} - \dfrac{\cos x}{2x}\right) \end{bmatrix}$$

Určete f´(-2) + f´(0) funkce:

$$a)\, f(x) = (x^3 + 2)^2 (x+1)$$

$$b)\, f(x) = e^{(x^2+3x)} \cdot 5 \cdot (x+2)$$

$$\begin{bmatrix} a) 180 + 4 = 184 \\ b) 5e^{-2} + 35 \end{bmatrix}$$

2. Derivujte podle x funkce tvaru $y = p(x)^{q(x)}$

(a) $y = (x+1)^x$ (b) $y = x^{x+1}$ (c) $y = x^{\sqrt{x}}$

(d) $y = (\cos x)^{\cos x}$ (e) $y = x^{\operatorname{tg} x}$ (f) $y = (\ln x)^{\ln x}$

Řešení:

2. (a) $y' = (x+1)^x(\ln(x+1) + \frac{x}{x+1})$; (b) $y' = x^{x+1}(\ln x + \frac{x+1}{x})$; (c) $y' = x^{\sqrt{x}} \cdot \frac{\ln x + 2}{2\sqrt{x}}$;
(d) $y' = -\sin x \cdot (\cos x)^{\cos x}(1 + \ln(\cos x))$; (e) $y' = x^{\operatorname{tg} x}(\frac{\ln x}{\cos^2 x} + \frac{\operatorname{tg} x}{x})$; (f) $y' = (\ln x)^{\ln x} \cdot \frac{\ln(\ln x) + 1}{x}$

Zderivujte funkce:

(a) $f(x) = x^5 - 4x^3 + 2x - 3$;
(b) $f(x) = \frac{\pi}{x} + \ln 2$;
(c) $f(x) = 3x^{\frac{2}{3}} - 2x^{\frac{5}{2}} + x^{-2}$;
(d) $f(x) = x^2\sqrt[3]{x^2}$;
(e) $f(x) = \frac{x+1}{x-1}$;
(f) $f(x) = \dfrac{2x+3}{x^2-5x+5}$;
(g) $f(z) = \dfrac{1+\sqrt{z}}{1-\sqrt{z}}$;
(h) $f(x) = \frac{a}{\sqrt[3]{x^2}} - \frac{b}{x\sqrt[3]{x}}$, a, b konst.;
(i) $f(x) = 5\sin x + 3\cos x$;
(j) $f(x) = \operatorname{tg} x - \operatorname{cotg} x$;
(k) $f(t) = 2t\sin t - (t^2-2)\cos t$;
(l) $f(x) = x \arcsin x$;
(m) $f(x) = \frac{\sin x + \cos x}{\sin x - \cos x}$;
(n) $f(x) = \frac{\arccos x}{\arcsin x}$;
(o) $f(x) = x^7 \cdot e^x$;
(p) $f(x) = e^{-x} \operatorname{arctg} x$;
(q) $f(x) = x^3 \ln x - \frac{x^3}{3}$;
(r) $f(x) = \frac{1}{x} + 2\ln x - \frac{\ln x}{x}$;
(s) $f(x) = \ln x \log x - \ln a \log_a x$;
(t) $f(t) = 5^t \operatorname{tg} t$;

Výsledek:

(a) $5x^4 - 12x^2 + 2$;
(b) $-\frac{\pi}{x^2}$, $x \neq 0$;
(c) $2x^{-\frac{1}{3}} - 5x^{\frac{3}{2}} - 2x^{-3}$, $x \neq 0$;
(d) $\frac{8}{3}x^{\frac{5}{3}}$;
(e) $-\frac{2}{(x-1)^2}$, $x \neq 1$;
(f) $\frac{-2x^2-6x+25}{(x^2-5x+5)^2}$, $x \neq \frac{5\pm\sqrt{5}}{2}$;
(g) $\frac{1}{\sqrt{z}(1-\sqrt{z})^2}$, $z > 0, z \neq 1$;
(h) $\frac{4b}{3x^2\sqrt[3]{x}} - \frac{2a}{3x\sqrt[3]{x^2}}$, $x \neq 0$;
(i) $5\cos x - 3\sin x$;
(j) $\frac{4}{\sin^2 2x}$, $x \neq k\frac{\pi}{2}$;
(k) $t^2 \sin t$;
(l) $\arcsin x + \frac{x}{\sqrt{1-x^2}}$, $x \in (-1, 1)$;
(m) $\frac{-2}{(\sin x - \cos x)^2}$, $x \neq \frac{\pi}{4} + k\pi$;
(n) $-\frac{\pi}{2\sqrt{1-x^2}\arcsin^2 x}$, $x \in (-1, 0) \cup (0, 1)$;
(o) $x^6 e^x (x+7)$;
(p) $e^{-x}\left(\frac{1}{1+x^2} - \operatorname{arctg} x\right)$;
(q) $3x^2 \ln x$, $x > 0$;
(r) $\frac{2}{x} + \frac{\ln x}{x^2} - \frac{2}{x^2}$, $x > 0$;
(s) $\dfrac{2\ln x}{x \ln 10} - \dfrac{1}{x}$, $x > 0$;
(t) $\dfrac{5^t(\ln 5 \sin 2t + 2)}{2\cos^2 t}$, $t \neq (2k+1)\frac{\pi}{2}$, $k \in \mathbf{Z}$;