Asymptoty

Určete asymptoty grafů funkcí:

a) $f(x)=\dfrac{x}{x-1}$

b) $f(x)=\dfrac{8}{4-x^2}$

c) $f(x)=\dfrac{x^2-3}{2x-4}$

d) $f(x)=\dfrac{x^2+1}{x+3}$

e) $f(x)=\dfrac{x^2-4}{x-1}$

f) $f(x)=\dfrac{x^2-1}{2x+4}$

g) $f(x)=\dfrac{4}{x^2-4}$

h) $f(x)=x+\dfrac{1}{x}$

i) $f(x)=\dfrac{x-2}{x+1}$

j) $f(x)=\dfrac{x^3}{x^2-1}$

Řešení:

a) x = 1, y = 1
b) x = -2, x = 2, y = 0
c) x = 2, y = 0,5x + 1
d) x = -3, y = x - 3
e) x = 1, y = x + 1
f) x = -2, y = 0,5x - 1
g) x = 2, x = -2, y = 0
h) x = 0, y = x
i) x = -1, y = 1
j) x = 1, x = -1, y = x

ⓐ bez směrnice $D_f = \mathbb{R} \setminus \{1\}$

$$\lim_{x \to 1^+} \frac{x}{x-1} = \left[\frac{1}{0^+}\right] = +\infty \checkmark \Rightarrow \underline{\underline{x = 1}}$$

se směrnicí $y = kx + q$

$$k = \lim_{x \to \pm\infty} \frac{x}{x^2 - x} = 0$$

$$q = \lim_{x \to \pm\infty} \frac{x}{x-1} - 0 = 1$$

$$\Big\} \ \underline{\underline{y = 1}}$$

ⓑ bez směrnice $D_f = \mathbb{R} \setminus \{\pm 2\}$

$$\lim_{x \to 2^+} \frac{8}{4 - x^2} = \left[\frac{8}{0^-}\right] = -\infty \Rightarrow \underline{\underline{x = 2}}$$

$$\lim_{x \to -2^+} \frac{8}{4 - x^2} = \left[\frac{8}{0^+}\right] = +\infty \Rightarrow \underline{\underline{x = -2}}$$

se směrnicí

$$k = \lim_{x \to \pm\infty} \frac{8}{4x - x^3} = 0$$

$$q = \lim_{x \to \pm\infty} \frac{8}{4 - x^2} - 0 = 0$$

$$\Big\} \ \underline{\underline{y = 0}}$$

ⓒ bez směrnice $D_f = \mathbb{R} \setminus \{2\}$

$$\lim_{x \to 2^+} \frac{x^2 - 3}{2x - 4} = \left[\frac{1}{0^+}\right] = +\infty \Rightarrow \underline{\underline{x = 2}}$$

se směrnicí

$$k = \lim_{x \to \pm\infty} \frac{x^2 - 3}{2x^2 - 4x} = \frac{1}{2}$$

$$q = \lim_{x \to \pm\infty} \frac{x^2 - 3}{2x - 4} - \frac{1}{2}x = \lim_{x \to \pm\infty} \frac{x^2 - 3 - x(x-2)}{2(x-2)} =$$

$$= \lim_{x \to \pm\infty} \frac{x^2 - 3 - x^2 + 2x}{2x - 4} = \lim_{x \to \pm\infty} \frac{2x - 3}{2x - 4} = 1$$

$$\underline{\underline{y = \frac{1}{2}x + 1}}$$