

LIMITY

$$1) \lim_{n \rightarrow \infty} \frac{2n^3 + 5n - 1}{4n^3 + 2n - 5}$$

$$2) \lim_{n \rightarrow \infty} \frac{2n^2 + 5n}{n^4 - 2n + 1}$$

$$3) \lim_{n \rightarrow \infty} \frac{3n^3 - 2n + 1}{-2n^2 + n + 5}$$

$$4) \lim_{n \rightarrow \infty} \frac{-n^3 - 5n + 1}{-2n + 2}$$

$$5) \lim_{n \rightarrow \infty} \frac{5n - 4}{2n + 2}$$

$$6) \lim_{n \rightarrow \infty} \frac{n^2 + 5n - 1}{4n^5 - 2n + 2}$$

$$7) \lim_{x \rightarrow 3} \frac{x^2 + 3}{x + 3}$$

$$8) \lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$$

$$9) \lim_{x \rightarrow 3} \frac{x^2 - 6x + 9}{81 - x^4}$$

$$10) \lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{x^2 - 8x + 15}$$

$$11) \lim_{x \rightarrow -2} \frac{x^2 - x - 6}{x^3 + 6x^2 + 8x}$$

$$12) \lim_{x \rightarrow 2} \frac{x + 1}{x - 2}$$

$$13) \lim_{x \rightarrow 3} \frac{x}{(x - 3)^2}$$

$$14) \lim_{x \rightarrow -1} \frac{2x}{(x + 1)(x - 2)}$$

$$15) \lim_{x \rightarrow 5} \frac{2x + 1}{x^2 - 25}$$

$$16) \lim_{x \rightarrow 1} \frac{-(x + 1)}{(x - 1)^2}$$

$$17) \lim_{x \rightarrow 0} \frac{x}{\sqrt{x + 9} - 3}$$

$$18) \lim_{x \rightarrow -2} \frac{2 - \sqrt{6 + x}}{x + 2}$$

LIMITY

- 1) $\lim_{n \rightarrow \infty} \frac{2n^3 + 5n - 1}{4n^3 + 2n - 5} = \frac{\frac{2}{4}}{\frac{4}{4}} = \frac{1}{2}$ (stejné mocnin)
- 2) $\lim_{n \rightarrow \infty} \frac{2n^2 + 5n}{n^4 - 2n + 1} = 0$ (některá mocnina je jmenovatele)
- 3) $\lim_{n \rightarrow \infty} \frac{3n^3 - 2n + 1}{-2n^2 + n + 5} = -\infty$ (některá mocnina v čitateli) $\frac{3n^3}{-2n^2} = \frac{3n}{-2} = \frac{3 \cdot \infty}{-2}$
- 4) $\lim_{n \rightarrow \infty} \frac{-n^3 - 5n + 1}{-2n + 2} = +\infty$ (některá mocnina v čitateli) $\frac{-n^3}{-2n} = \frac{n^2}{2}$
- 5) $\lim_{n \rightarrow \infty} \frac{5n - 4}{2n + 2} = \frac{5}{2}$ (stejné mocnin)
- 6) $\lim_{n \rightarrow \infty} \frac{n^2 + 5n - 1}{4n^5 - 2n + 2} = 0$ (některá je jmenovatele)
- 7) $\lim_{x \rightarrow 3} \frac{x^2 + 3}{x + 3} = \frac{9+3}{3+3} = \frac{12}{6} = 2$
- 8) $\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = \left[\frac{9-9}{3-3} \right] = \left[\frac{0}{0} \right] \stackrel{\text{L'H}}{\lim_{x \rightarrow 3}} \frac{2x}{1} = 6$
- 9) $\lim_{x \rightarrow 3} \frac{x^2 - 6x + 9}{81 - x^4} = \left[\frac{0}{0} \right] \stackrel{\text{L'H}}{\lim_{x \rightarrow 3}} \frac{2x - 6}{-4x^3} = \frac{0}{-4 \cdot 27} = 0$
- 10) $\lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{x^2 - 8x + 15} = \left[\frac{0}{0} \right] \stackrel{\text{L'H}}{\lim_{x \rightarrow 3}} \frac{2x + 2}{2x - 8} = \frac{8}{-2} = -4$
- 11) $\lim_{x \rightarrow -2} \frac{x^2 - x - 6}{x^3 + 6x^2 + 8x} = \left[\frac{0}{0} \right] \stackrel{\text{L'H}}{\lim_{x \rightarrow -2}} \frac{2x - 1}{3x^2 + 12x + 8} = \frac{-5}{-4} = \frac{5}{4}$
- 12) $\lim_{x \rightarrow 2} \frac{x+1}{x-2} = \left[\frac{3}{0} \right] \rightarrow \text{jednoz. lim.}$
 $\lim_{x \rightarrow 2^+} \frac{x+1}{x-2} = \left[\frac{3}{0^+} \right] = +\infty$
 $\lim_{x \rightarrow 2^-} \frac{x+1}{x-2} = \left[\frac{3}{0^-} \right] = -\infty$ } necht.
- 13) $\lim_{x \rightarrow 3} \frac{x}{(x-3)^2} = \left[\frac{3}{0} \right] \rightarrow \begin{aligned} \lim_{x \rightarrow 3^+} \frac{x}{(x-3)^2} &= \left[\frac{3}{0^+} \right] = +\infty \\ \lim_{x \rightarrow 3^-} \frac{x}{(x-3)^2} &= \left[\frac{3}{0^+} \right] = +\infty \end{aligned}$
 $\Rightarrow \text{jedn. slodek} = +\infty$
- 14) $\lim_{x \rightarrow -1} \frac{2x}{(x+1)(x-2)} \quad \text{(14)}$
 $\lim_{x \rightarrow -1} \frac{2x}{(x+1)(x-2)} = \left[\frac{-2}{0} \right] = \text{necht.}$
 $\lim_{x \rightarrow -1^+} \frac{2x}{(x+1)(x-2)} = \left[\frac{-2}{0^+(-3)} \right] = +\infty$
 $\lim_{x \rightarrow -1^-} \frac{2x}{(x+1)(x-2)} = \left[\frac{-2}{0^-(3)} \right] = -\infty$
- 15) $\lim_{x \rightarrow 5} \frac{2x+1}{x^2 - 25}$
 $\lim_{x \rightarrow 5^+} \frac{2x+1}{x^2 - 25} = \left[\frac{11}{0^+} \right] = \text{nechtislož.}$
 $\lim_{x \rightarrow 5^-} \frac{2x+1}{x^2 - 25} = \left[\frac{11}{0^-} \right] = -\infty$
- 16) $\lim_{x \rightarrow 1} \frac{-(x+1)}{(x-1)^2} = \left[\frac{-2}{0} \right] = -\infty$
 $\lim_{x \rightarrow 1^+} \frac{-(x+1)}{(x-1)^2} = \left[\frac{-2}{0^+} \right] = -\infty$
 $\lim_{x \rightarrow 1^-} \frac{-(x+1)}{(x-1)^2} = \left[\frac{-2}{0^-} \right] = -\infty$
- 17) $\lim_{x \rightarrow 0} \frac{x}{\sqrt{x+9} - 3}$
 $\boxed{\text{(15)} \lim_{x \rightarrow 5} \frac{2x+1}{x^2 - 25} = \left[\frac{11}{0} \right] = \text{nechtislož.}}$
- 18) $\lim_{x \rightarrow -2} \frac{2 - \sqrt{6+x}}{x+2}$
 $\lim_{x \rightarrow 5^+} \frac{2x+1}{x^2 - 25} = \left[\frac{11}{0^+} \right] = +\infty$
 $\lim_{x \rightarrow 5^-} \frac{2x+1}{x^2 - 25} = \left[\frac{11}{0^-} \right] = -\infty$
- (16) $\lim_{x \rightarrow 1} \frac{-(x+1)}{(x-1)^2} = \left[\frac{-2}{0} \right] = -\infty$
 $\lim_{x \rightarrow 1^+} \frac{-(x+1)}{(x-1)^2} = \left[\frac{-2}{0^+} \right] = -\infty$
 $\lim_{x \rightarrow 1^-} \frac{-(x+1)}{(x-1)^2} = \left[\frac{-2}{0^-} \right] = -\infty$
- (17) $\lim_{x \rightarrow 0} \frac{x}{\sqrt{x+9} - 3} = \left[\frac{0}{0} \right] \stackrel{\text{L'H}}{\lim_{x \rightarrow 0}} \frac{1}{\frac{1}{2\sqrt{x+9}}} = \frac{1}{\frac{1}{2 \cdot 3}} = 6$
- (18) $\lim_{x \rightarrow -2} \frac{2 - \sqrt{6+x}}{x+2} = \left[\frac{0}{0} \right] \stackrel{\text{L'H}}{\lim_{x \rightarrow -2}} \frac{-\frac{1}{2\sqrt{6+x}}}{1} = \frac{-1}{2\sqrt{4}} = -\frac{1}{4}$

Vypočítejte limity:

a) $\lim_{x \rightarrow \infty} \frac{2x^2 + x}{-3x^2 + 5x - 1}$

b) $\lim_{x \rightarrow 0} \frac{x^2 + 2x - 1}{x^2 + x + 1}$

c) $\lim_{x \rightarrow 10} \frac{\sqrt{x-1} - 3}{x-10}$

d) $\lim_{x \rightarrow -\infty} \frac{x^2 + x + 3}{3x^3 + 2x}$

e) $\lim_{x \rightarrow -1} \frac{x^3}{x^2 - 1}$

f) $\lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{2x^2 - x - 1}$

g) $\lim_{x \rightarrow -2} \frac{2 - \sqrt{6+x}}{x+2}$

h) $\lim_{x \rightarrow -\infty} \frac{-2x^3 + 4x - 1}{2x^2 + x}$

i) $\lim_{x \rightarrow -1} \frac{x^3 - 3x - 2}{x^2 - x - 2}$

j) $\lim_{x \rightarrow 1} \frac{2 - \sqrt{x+3}}{\sqrt{x}-1}$

k) $\lim_{x \rightarrow \infty} \frac{-x^4 + x}{x^2 + x}$

l) $\lim_{x \rightarrow -2} \frac{x-2}{x^2 + 4x + 4}$

m) $\lim_{x \rightarrow 3} \frac{9 - x^2}{\sqrt{3x} - 3}$

n) $\lim_{x \rightarrow 3} \frac{\sqrt{x+13} - 2\sqrt{x+1}}{x^2 - 9}$

o) $\lim_{x \rightarrow 3} \frac{x+1}{(x-3)^3}$

p) $\lim_{x \rightarrow 0} \frac{\sqrt{1-2x-x^2} - (1+x)}{x}$

q) $\lim_{x \rightarrow 1} \frac{x^3 - 3x + 2}{x^4 - 4x + 3}$

r) $\lim_{x \rightarrow 2} \frac{x^4 - 2x^3 + 2x^2 - 5x + 2}{x-2}$

s) $\lim_{x \rightarrow 0} \frac{\sqrt{(1+x)^3} - 1}{x}$

Řešení:

- a) -2/3 b) -1 c) 1/6 d) 0 e) neexistuje f) 0 g) -1/4 h) $+\infty$ i) 0 j) -1/2 k) $-\infty$ l) $-\infty$ m) -12
- n) -1/16 o) neexistuje p)-2 q) $\frac{1}{2}$ r) 11 s) 3/2