Vypočtěte následující limity pomocí l'Hopitalova pravidla:

Typ $\frac{0}{0}$ a $\frac{\pm\infty}{\pm\infty}$.

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1. $\lim\limits_{x\to\infty} \frac{x\ln x}{x^2+x+1} = ||\tfrac{\infty}{\infty}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to\infty} \frac{\ln x+1}{2x+1} ||\tfrac{\infty}{\infty}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to\infty} \frac{\frac{1}{x}}{2} \overset{\text{dosadit}}{=} 0$

2. $\lim\limits_{x\to 0} \frac{\operatorname{arctg} 2x}{\arcsin x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \frac{\frac{2}{1+4x^2}}{\frac{1}{\sqrt{1-x^2}}} \overset{\text{uprava}}{=} \lim\limits_{x\to 0} \frac{2\sqrt{1-x^2}}{1+4x^2} \overset{\text{dosadit}}{=} 2$

3. $\lim\limits_{x\to 1} \frac{x-1}{\ln x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 1} \frac{1}{\frac{1}{x}} \overset{\text{dosadit}}{=} 1$

4. $\lim\limits_{x\to\infty} \frac{\ln x}{\sqrt{x}} = ||\tfrac{\infty}{\infty}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = ||\tfrac{0}{0}|| \overset{\text{uprava}}{=} \frac{1}{2} \lim\limits_{x\to\infty} \frac{1}{\sqrt{x}} = 0$

5. $\lim\limits_{x\to 0} \frac{\arcsin x}{1-e^x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{-e^x} \overset{\text{dosadit}}{=} -1$

6. $\lim\limits_{x\to e} \frac{\ln x-1}{\ln^2 x+\ln x-2} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to e} \frac{\frac{1}{x}}{2\frac{1}{x}\ln x+\frac{1}{x}} \overset{\text{uprava}}{=} \lim\limits_{x\to e} \frac{1}{2\ln x+1} \overset{\text{dosadit}}{=} \frac{1}{3}$

7. $\lim\limits_{x\to 1} \frac{\ln x}{\operatorname{cotg} \frac{x\pi}{2}} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 1} \frac{\frac{1}{x}}{-\frac{1}{\sin^2\frac{\pi x}{2}}\frac{\pi}{2}} \overset{\text{dosadit}}{=} -\frac{2}{\pi}$

8. $\lim\limits_{x\to 0^+} \frac{\ln(e^x-1)}{\ln x} = ||\tfrac{\infty}{\infty}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0^+} \frac{\frac{1}{e^x-1}e^x}{\frac{1}{x}} \overset{\text{uprava}}{=} \lim\limits_{x\to 0^+} \frac{xe^x}{e^x-1} ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0^+} \frac{e^x+xe^x}{e^x} \overset{\text{dosadit}}{=} 1$

9. $\lim\limits_{x\to 0} \frac{e^x-1}{\sin 2x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \frac{e^x}{2\cos 2x} \overset{\text{dosadit}}{=} \frac{1}{2}$

10. $\lim\limits_{x\to 0} \frac{x-\sin x}{\sin^3 x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \frac{1-\cos x}{3\sin^2 x\cos x} ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \frac{\sin x}{6\sin x\cos^2 x-3\sin^3 x} ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=}$

$= \lim\limits_{x\to 0} \frac{\cos x}{6\cos^3 x-6\cdot 2\cdot\sin^2 x\cos x-9\sin^2 x\cos x} \overset{\text{dosadit}}{=} \frac{1}{6}$

11. $\lim\limits_{x\to 3} \frac{x-3}{x^2-8x+15} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 3} \frac{1}{2x-8} \overset{\text{dosadit}}{=} -\frac{1}{2}$

12. $\lim\limits_{x\to 1} \frac{1+\cos\pi x}{(x-1)^2} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 1} \frac{-\pi\sin\pi x}{2(x-1)} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 1} \frac{-\pi^2\cos\pi x}{2} \overset{\text{dosadit}}{=} \frac{\pi^2}{2}$

13. $\lim\limits_{x\to\infty} \frac{x^2}{e^{x^2}} = ||\tfrac{\infty}{\infty}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to\infty} \frac{2x}{2xe^{x^2}} \overset{\text{uprava}}{=} \lim\limits_{x\to\infty} \frac{1}{e^{x^2}} \overset{\text{dosadit}}{=} 0$

14. $\lim\limits_{x\to 1} \frac{1-x}{\operatorname{cotg}\frac{\pi x}{2}} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 1} \frac{-1}{-\frac{\pi}{2}\sin^{-2}\frac{\pi x}{2}} \overset{\text{dosadit}}{=} \frac{2}{\pi}$

15. $\lim\limits_{x\to 0} \frac{\sin x-x}{x\sin x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \frac{\cos x-1}{\sin x+x\cos x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \frac{-\sin x}{\cos x+\cos x-x\sin x} \overset{\text{dosadit}}{=} 0$

16. $\lim\limits_{x\to\frac{\pi}{2}} \frac{\operatorname{tg} 3x}{\operatorname{tg} x} = ||\tfrac{\infty}{\infty}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to\frac{\pi}{2}} \frac{\frac{3}{\cos^2 3x}}{\frac{1}{\cos^2 x}} = ||\tfrac{\infty}{\infty}|| \overset{\text{uprava}}{=} 3\lim\limits_{x\to\frac{\pi}{2}} \frac{\cos^2 x}{\cos^2 3x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} 3\lim\limits_{x\to\frac{\pi}{2}} \frac{-2\sin x\cos x}{-6\sin 3x\cos 3x} = ||\tfrac{0}{0}||$

$\overset{\text{uprava}}{=} 3\cdot\frac{2}{6}\cdot\lim\limits_{x\to\frac{\pi}{2}} \frac{\sin x}{\sin 3x}\cdot\lim\limits_{x\to\frac{\pi}{2}} \frac{\cos x}{\cos 3x} \overset{\text{uprava}}{=} -\lim\limits_{x\to\frac{\pi}{2}} \frac{\cos x}{\cos 3x} = ||\tfrac{0}{0}|| \overset{\text{l'H.p.}}{=} -\lim\limits_{x\to\frac{\pi}{2}} \frac{-\sin x}{-3\sin 3x} \overset{\text{dosadit}}{=} \frac{1}{3}$

16B. $\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\operatorname{tg} 3x}{\operatorname{tg} x} = ||\frac{\infty}{\infty}|| \overset{\text{uprava}}{=} \lim\limits_{x\to\frac{\pi}{2}} \dfrac{\sin 3x \cos x}{\sin x \cos 3x} \overset{\text{uprava}}{=} \lim\limits_{x\to\frac{\pi}{2}} \dfrac{\sin 3x}{\sin x} \cdot \lim\limits_{x\to\frac{\pi}{2}} \dfrac{\cos x}{\cos 3x} = -\lim\limits_{x\to\frac{\pi}{2}} \dfrac{\cos x}{\cos 3x} = ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=}$
$= -\lim\limits_{x\to\frac{\pi}{2}} \dfrac{-\sin x}{-3\sin 3x} \overset{\text{dosadit}}{=} \dfrac{1}{3}$

17. $\lim\limits_{x\to 0} \dfrac{\boldsymbol{x - \operatorname{arctg} x}}{\boldsymbol{x^3}} = ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \dfrac{1 - \frac{1}{1+x^2}}{3x^2} = ||\frac{0}{0}|| \overset{\text{uprava}}{=} \lim\limits_{x\to 0} \dfrac{1}{3(1+x^2)} \overset{\text{dosadit}}{=} \dfrac{1}{3}$

18. $\lim\limits_{x\to 0} \dfrac{\boldsymbol{xe^x + x - 2e^x + 2}}{\boldsymbol{x^3}} = ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \dfrac{e^x + xe^x + 1 - 2e^x}{3x^2} = ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \dfrac{e^x + e^x + xe^x - 2e^x}{6x} \overset{\text{uprava}}{=}$
$= \lim\limits_{x\to 0} \dfrac{xe^x}{6x} \overset{\text{uprava}}{=} \lim\limits_{x\to 0} \dfrac{e^x}{6} \overset{\text{dosadit}}{=} \dfrac{1}{6}$

19. $\lim\limits_{x\to 0} \dfrac{\boldsymbol{xe^x + x - 2e^{-x} + 2}}{\boldsymbol{x^3}} = ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \dfrac{e^x + xe^x + 1 + 2e^x}{3x^2} \overset{\text{dosadit}}{=} \dfrac{4}{+0} = \infty$

20. $\lim\limits_{x\to 0} \dfrac{\boldsymbol{\cos 2x - 1}}{\boldsymbol{2\sin^2 x + 2x \sin 2x}} = ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \dfrac{-2\sin 2x}{4\sin x \cos x + 2\sin 2x + 4x\cos 2x} = ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=}$
$= \lim\limits_{x\to 0} \dfrac{-4\cos 2x}{4\cos x \cos x - 4\sin x \sin x + 4\cos 2x + 4\cos 2x - 8x\sin 2x} \overset{\text{dosadit}}{=} -\dfrac{1}{3}$

21. $\lim\limits_{x\to 0} \dfrac{\boldsymbol{e^x - e^{-x} - 2x}}{\boldsymbol{x - \sin x}} = ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \dfrac{e^x + e^{-x} - 2}{1 - \cos x} = ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \dfrac{e^x - e^{-x}}{\sin x} = ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=}$
$= \lim\limits_{x\to 0} \dfrac{e^x + e^{-x}}{\cos x} \overset{\text{dosadit}}{=} 2$

22. $\lim\limits_{x\to 0} \dfrac{\boldsymbol{1 - \cos x^2}}{\boldsymbol{x^2 \sin x^2}} = ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \dfrac{2x\sin x^2}{2x\sin x^2 + 2xx^2\cos x^2} \overset{\text{uprava}}{=}$
$= \lim\limits_{x\to 0} \dfrac{\sin x^2}{\sin x^2 + x^2\cos x^2} = ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \dfrac{2x\cos x^2}{2x\cos x^2 + 2x\cos x^2 - 2xx^2\sin x^2} = ||\frac{0}{0}|| \overset{\text{uprava}}{=}$
$= \lim\limits_{x\to 0} \dfrac{\cos x^2}{\cos x^2 + \cos x^2 - x^2\sin x^2} \overset{\text{dosadit}}{=} \dfrac{1}{2}$

23. $\lim\limits_{x\to 0} \dfrac{\boldsymbol{x \operatorname{cotg} x - 1}}{\boldsymbol{x^2}} = \lim\limits_{x\to 0} \dfrac{x\cos x - \sin x}{x^2 \sin x} = ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \dfrac{\cos x - x\sin x - \cos x}{2x\sin x + x^2\cos x} \overset{\text{uprava}}{=}$
$= -\lim\limits_{x\to 0} \dfrac{\sin x}{2\sin x + x\cos x} ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \dfrac{\cos x}{2\cos x + \cos x - x\sin x} \overset{\text{dosadit}}{=} -\dfrac{1}{3}$

24. $\lim\limits_{x\to 0} \dfrac{\boldsymbol{x\cos x - \sin x}}{\boldsymbol{x \sin x}} = ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \dfrac{\cos x - x\sin x - \cos x}{\sin x + x\cos x} \overset{\text{uprava}}{=} -\lim\limits_{x\to 0} \dfrac{-x\sin x}{\sin x + x\cos x} = ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=}$
$= -\lim\limits_{x\to 0} \dfrac{\sin x + x\cos x}{\cos x + \cos x - x\sin x} \overset{\text{dosadit}}{=} 0$

25. $\lim\limits_{x\to 0^+} \dfrac{\boldsymbol{e^x - 1 - x}}{\boldsymbol{(e^x - 1)x}} = ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0^+} \dfrac{e^x - 1}{e^x - 1 - xe^x} = ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0^+} \dfrac{e^x}{e^x - xe^x - e^x} \overset{\text{dosadit}}{=} \dfrac{1}{2}$

26. $\lim\limits_{x\to 1} \dfrac{\boldsymbol{x\ln x - x + 1}}{\boldsymbol{(x-1)\ln x}} = ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 1} \dfrac{\ln x + x\frac{1}{x} - 1}{\frac{x-1}{x} + \ln x} \overset{\text{uprava}}{=} \lim\limits_{x\to 1} \dfrac{\ln x}{\frac{x-1}{x} + \ln x} \overset{\text{uprava}}{=} \lim\limits_{x\to 1} \dfrac{x\ln x}{(x-1) + x\ln x} =$
$= ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 1} \dfrac{1 + \ln x}{2 + \ln x} \overset{\text{dosadit}}{=} \dfrac{1}{2}$

27. $\lim\limits_{x\to 0} \dfrac{\boldsymbol{x - \sin x}}{\boldsymbol{\sin^3 x}} = ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \dfrac{1 - \cos x}{3\sin^2 x \cos x} ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=} \lim\limits_{x\to 0} \dfrac{\sin x}{6\sin x\cos^2 x - 3\sin^3 x} ||\frac{0}{0}|| \overset{\text{l'H.p.}}{=}$
$= \lim\limits_{x\to 0} \dfrac{\cos x}{6\cos^3 x - 6\cdot 2\cdot \sin^2 x\cos x - 9\sin^2 x\cos x} \overset{\text{dosadit}}{=} \dfrac{1}{6}$